Identification of the optimal racing line for top male athletes on a segment of a World Cup downhill alpine ski slope using tracking data

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ABSTRACT
Global-Navigation-Satellite-System-tracking data (GNSS) are used in alpine skiing to investigate athlete's speed and acceleration as a standard. In this paper, we present a case study, in which we use GNSS-tracking data to identify additionally the racing line with the optimal geometric shape among 10 different racing lines chosen by elite athletes. For this, we concentrate on the famous S-curved segment (Brüggli-S) of the downhill FIS World Cup track Lauberhorn in Switzerland. We have measured and analyzed training runs, measured on a single day, of 10 world-class athletes. The geometric shapes of each measured racing line share a similar pattern: The maximum curvature in the first curve is strongly correlated with the maximum curvature of the second curve. For the athletes under investigation, the overall best racing line is characterized by a defined ratio of the maximum curvature of the first to the second curve. We suggest that the observed correlation, as well as the optimal ratio of curvatures, is universal for a class of athletes with nearly identical performance levels. If and how this ratio varies for different classes of athletes (i.e. U10, U12 or U16 athletes) is an as yet unanswered question and remains open to further investigation.

Keywords
optimal racing line, alpine skiing, downhill, GNSS, performance analysis, elite sports
Introduction

The four main competition disciplines of the alpine ski racing World Cup are slalom, giant slalom, super-G and downhill. Each race has its own specific course and terrain characteristics regulated by the International Ski Federation (FIS). During the race, there are the skiers’ speed, the skiers’ body movement and the chosen path (racing line), which influences the performance (Luginbühl et al., 2023).

For performance analysis in alpine ski racing, video analysis is a standard. Additionally, in the past years Global Navigation Satellite Systems (GNSS) sensors became popular for analyzing alpine ski racing. Acceleration and velocity data on a given racing line derived from GNSS-tracking data are used as important performance indicators and delivered important insights into alpine ski racing (Bruhin et al., 2020; Fasel et al., 2016; Gilgien et al., 2013, 2015; Spörri, 2012).

Beyond acceleration and velocity, though, the geometric shape of the racing line as chosen by the athlete has a major impact on the performance. In this paper we analyze the geometric shape of the racing line of the S-curved segment (Brüggli-S) of the Lauberhorn track. The aim of this paper is to show how GNSS-data can also be used to define performance indicators for the quality of the shape of a racing line. On this basis, we have used a quality measure for the investigated segment of the racing line, allowing us to distinguish between bad and good racing lines, and even identify the optimal racing line on the basis of tracking data. If and how, this work can be generalized is an open question and subject of future research.

One problem of GNSS measurements is the necessary compromise between accuracy of the GNSS-tracking data and the portability of the equipment. Since the more accurate, differential GNSS systems require more equipment, these are not worn in training or race situations (Gilgien et al., 2015). In training or race situations, stand-alone GNSS sensors have become small enough to wear (Bruhin et al., 2020; Fasel et al., 2016). There are some concepts which combine GNSS information with IMU data, enhancing the accuracy of the sensor information (Fasel et al., 2016; Yu et al., 2016).

Methods

Participants, material & experimental setup

Measurements were taken for ten male elite skiers from the Swiss National team of mean age (27.6 ± 3.7) years, and had a mean world ranking of 61.0 ± 50.9 (min.: 4; max.: 187), based on downhill FIS points. The measurements took place during one training run on the Lauberhorn World Cup slope on January 14th, 2020 in Wengen, Switzerland. We consider the group as homogenous as such:

a. the variation in technical ability in downhill skiing of the athletes is small,

b. the variation in material used by the athletes is small, and

c. there was only a small change in environmental conditions (temperature, snow, humidity, and others).

An underlying assumption of our analysis is, that for the segment of the racing line (Brüggli-S), the ski-technical skills required to perform the racing line are
on the upper edge of the athlete’s skill level. Therefore, racing lines, which are dominated by a ski-technical fault of the athlete in the Brüggli-S segment are excluded from the analysis. By video analysis, we have confirmed that in the segment the athletes did not suffer from such serious technical problems.

Each athlete wore a lightweight (35 g) portable GNSS sensor (FieldWiz, developed and manufactured by ASI in Switzerland) on their spinal protectors. The sensor was attached to the back protector with double-sided adhesive tape at the approximate height of the third thoracic vertebrae. For safety reasons, it was placed to the left of the spine. The sensor delivered time-discrete 3-dimensional data \( \hat{\mathbf{s}}(t_i) = [s_x(t_i), s_y(t_i), s_z(t_i)] \) with a time interval \( \Delta t = 0.1 \) s (10 Hz). For this study, the 10 skiers skied the entire downhill FIS World Cup track Lauberhorn in Switzerland. For the performance analysis of the athletes, the entire run was analyzed. In this paper though, to address the particular research question, only the Brüggli-S segments were extracted from the measurements. Therefore, unless otherwise stated, the data throughout this paper corresponds to the Brüggli-S segment.

**Time series analysis**

In this paper, we consider the continuous, time-dependent 3-dimensional vector \( \mathbf{r}(t) \) of the athlete’s position and its derivatives, the 3-dimensional velocity of the athlete \( \mathbf{v}(t) = \frac{d\mathbf{r}(t)}{dt} \), and the 3-dimensional acceleration \( \mathbf{a}(t) = \frac{d\mathbf{v}(t)}{dt} \) as the athlete’s racing line. The unknown time-continuous racing line \( \mathbf{r}(t), \mathbf{v}(t), \mathbf{a}(t) \) is estimated from the time-discrete tracking data \( \mathbf{s}(t_i) = [s_x(t_i), s_y(t_i), s_z(t_i)] \) with the help of time series techniques (see section below). Until very recently, the analysis of an athlete’s performance, or the comparison of performances of several athletes, relied on highly subjective measures, such as derivation from video study. The racing line encompasses (a) the geometric shape of the athlete’s movement in the 3-dimensional space, and (b) the athlete’s dynamics (velocities and accelerations) as he follows the path. An important first step in the analysis is to construct the time-continuous racing line \( [\mathbf{r}(t), \mathbf{v}(t), \mathbf{a}(t)] \) from the time-discrete tracking (location) measurement \( \mathbf{s}(t_i) \) with high accuracy. For this, we use a Savitzky-Golay-filter, which allows the estimation of:

a. 1st order derivatives,
b. 2nd order derivatives, and
c. the interpolation of data with high accuracy (Press et al., 1986).

The racing line contains important information, since it reflects:

a. cognitive elements, such as the athlete’s choice of how to direct his motion to optimally take advantage of the slopes delivered by the racing track,
b. the physics of the motion, how the racing track’s slopes accelerate the athlete and the friction forces the athlete experiences during the run, and
c. the technical capabilities of the athlete in handling the equipment and the environment.

In Figure 1(a) we present the birds-eye-view of the geometric shapes of the different racing lines \( \mathbf{r}(t) \) chosen by the athletes.

As the Savitzky-Golay filter is especially suited to estimating derivatives from discrete data (Press et al., 1986) we can estimate the 3-dimensional time-discrete velocity \( \mathbf{v}(t) = \frac{d\mathbf{r}(t)}{dt} = [v_x(t), v_y(t), v_z(t)] \) and the 3-dimensional time-discrete acceleration \( \mathbf{a}(t) = \frac{d\mathbf{v}(t)}{dt} = [a_x(t), a_y(t), a_z(t)] \) with a high level of accuracy. With this, the norm of the velocities, and the norm of the accelerations is easily computed:

\[ \|\mathbf{v}(t)\| = \sqrt{v_x^2(t) + v_y^2(t) + v_z^2(t)}, \quad \|\mathbf{a}(t)\| = \sqrt{a_x^2(t) + a_y^2(t) + a_z^2(t)}.\]

In Figure 1(b) and Figure 1(c) we present the norm of the velocities, and the norm of the accelerations as measured for the racing lines of the 10 athletes.
From this data, we conclude that – as expected – the dynamics of the athletes differ considerably.

On the basis of the 3-dimensional velocities and accelerations, the curvature of the racing line can be estimated for each athlete via:

\[ k(t) = \frac{\| \dot{\mathbf{r}}(t) \times \mathbf{r}(t) \|}{\| \mathbf{r}(t) \|}. \]

Loosely speaking, the curvature \( k(t) \) measures how fast the racing line changes direction at a given time. In Figure 2, we present the curvatures \( k(t) \) of the racing lines as measured for the 10 athletes. With the help of the curvatures, the differences in the geometric shapes of the racing lines of the 10 athletes is considerably more obvious compared to trajectory data in the birds-eye-view as shown in Figure 1(a). As such, the curvature can serve as a magnifying glass to investigate differences in the geometry of racing lines, and an athlete’s choices of how to master the S-curve.

**Figure 1** (a) Geometric shapes of the racing lines of the ten athletes in the segment Brüggli-S in the birds-eye-view. The 2 virtual light barriers L1 and L2 are also indicated. (b) Velocities of the racing lines of the ten athletes in the segment Brüggli-S. (c) Accelerations of the racing lines of the ten athletes in the segment Brüggli-S.
As of now, we have used time series analysis to uncover differences of the racing lines of the 10 athletes, (a) in terms of the dynamics, and (b) in terms of the geometry. In the following, we will show that these two are strongly correlated: The dynamics of the racing line defines its geometry – or the other way around – the geometry of the racing line defines its dynamics. Naturally, we will identify the optimal racing line as the one, which provides the best racing performance.

To this end, three virtual light barriers (L1, L2, and L3) are superimposed on the slope, see Figure 1(a) for the positioning of L1 and L2. The light barrier L3 is positioned 475 m behind L2. Using these, we can interpolate the time-discrete data and obtain the velocities \( v_1, v_2, v_3 \) and the time stamp \( t_1, t_2, t_3 \) of the athlete as he passes the light barrier. The total time difference between light barrier 1 and 3, we compute as \( T = t_3 - t_1 \).

Figure 2 Curvatures of the racing lines of the ten athletes in the segment Brüggli-S.

### Statistical analysis

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### Results

Let us now consider the velocities \( v_1 \) and \( v_2 \) as measured by the virtual light barriers L1 and L2. The velocity \( v_2 \) is the athlete’s velocity as he leaves the S-curve. Athletes try to leave the S-curve with as high a velocity \( v_2 \) as possible, since after the S-curve there is an extended gliding section and a high velocity \( v_2 \) will provide an advantage to the athlete. Therefore, \( v_2 \) can serve as a quality criterion for the athlete’s performance in the Brüggli-S section. In Figure 3(a) we show \( v_2 \) versus \( v_1 \) for all 10 athletes. We find that the velocity \( v_2 \) shows little to no correlation with \( v_1 \). As such, the velocity \( v_1 \) before the S-curve is not a predictor for the velocity \( v_2 \) at the exit of the S-curve.

As the athlete passes from L1 to L2, he first has to take a right curve and then a left curve. The maximum curvature \( k_1 \) of the first (right) curve is recorded, as well as the maximum curvature \( k_{II} \) of the second (left) curve for every athlete. The two maximum curvatures \( k_1 \) and \( k_{II} \) are the dominant quantities for the geo-
metric shapes of the athletes racing line. In Figure 3(b) we show $k_I$ vs. $k_{II}$. The 2 maximum curvatures are correlated and follow a linear law:

$$k_{II} = -1.74 k_I + 0.076$$

with $r = -0.9197$. As such, $k_I$ is a good predictor for $k_{II}$, and therefore, the geometric shape of the athlete’s racing line is defined up to a reasonable degree of accuracy by the maximal curvature of the first (right) curve only.

Next, we analyze how the exit velocity $v_2$ depends on the maximum curvatures of the racing line. Since the maximum curvatures of the first and second curve are closely related, we consider only the maximum curvature of the first curve $k_I$. In Figure 4(a) we show $v_2$ versus $k_I$ for all 10 athletes. From the measurement, a linear law can be extracted:

$$v_2 = 307 k_I + 13.8$$

with $r = 0.6909$. As such, we find the geometric shape of the racing line (the curvatures) chosen by the athletes is a good predictor for the exit velocity $v_2$. As after the Brüggli-S segment there is an extended straight segment with gliding, the speed $v_2$ needs to be as high as possible. Our analysis shows that the highest speed $v_2$ can be achieved with a high $k_I$, which is a characteristic of the geometric shape of the racing line. Therefore, in this case study, the value of $k_I$ can be used as a performance criterion for the geometric shape of the racing line.

To show how strongly the geometric shape of the racing line influences the timing of the athletes, we investigate the correlation between the time $T$ needed to proceed from the virtual light barrier $L_1$ to the virtual light barrier $L_3$, which lies 475 m behind the light barrier $L_2$. In Figure 4(b) we show $T$ versus $k_I$ for all 10 athletes. We see that the fastest athlete has chosen a racing line with high $k_I$. The smaller the value of $k_I$ adopted by the athlete, the more the athlete suffers from a larger $T$. From the measurement, a linear law can be extracted:

$$T = -89.8 k_I + 24.5$$
Figure 4 (a) Maximal curvature $k_j$ for the first curve of the Brüggli-S segment, and the exit velocity $v_2$ measured for the 10 athletes. (b) Maximal curvature $k_j$ for the first curve of the Brüggli-S segment, and time $T$ to go from the virtual light barrier L1 to the virtual light barrier L3 measured for the 10 athletes.

with $r = -0.4040$. The correlation between $v_2$ and $k_j$ is stronger than the correlation between $T$ and $k_j$, which is reasonable since, as the athlete passes from $L_2$ to $L_3$ (difference 475 m), the time $T$ is influenced by many other factors, diminishing the correlation. In the eyes of the authors, it is astonishing that the correlation remains as strong as reported. Therefore, the effect of the chosen racing line in the Brüggli-S segment has a great impact, not only on that section, but on the entire run.

Conclusions

We have found that for the 10 measured racing lines of 10 Swiss top male athletes, the maximum curvature of the first curve is a performance-criteria for the chosen racing line in the Brüggli-S segment. In the framework of the data the quality of the racing line increases as the curvature of the first curve increases. Clearly, this cannot be the full truth, since there must be a maximal value of the curvature of the first curve. To detect this optimal curvature was not able with the accessible data – it will require additional measurements.

But, to our knowledge, for the first time, we were able to deduce this result from measurements of racing lines of top male athletes. To achieve this, we used high-end motion tracking technology. Therefore, we have shown that modern tracking technology is a valid tool to further improve the fundamental understanding of skiing. For the future, we are highly confident that the tracking technology will be able to unveil new insights, which are less obvious, and maybe even counter-intuitive compared with today's experience-based knowledge.

For coaches and athletes, as GNSS-tracking with time series analysis is used in training, the proposed criteria will help to understand how the choice of the geometry of the racing line is connected to racing success. It will reveal necessary corrections of the geometry of
the racing line, and gives therefore valuable hints for improvement for athletes and coaches.

Discussion

In the field of GNSS-3D-tracking in downhill skiing, a proper balance between the weight of an athlete's measurement equipment with the accuracy of measurement has to be found. In this study we used a very small and lightweight sensor to disturb the athlete as little as possible, slightly compromising the accuracy of measurement. In this paper, we have demonstrated that, even in this experimental setup, with the proper use of time series analysis techniques, racing lines can be reconstructed from data with high accuracy.

This study and the derived performance criteria are specific to the Brüggli-S segment of the Lauberhorn racing track. In our eyes, it can be easily extended to other S-shaped racing track segments. The generalization of this work to more extended segments of a racing track or complete racing tracks is an open question.

It is a well-accepted truth among top coaches and athletes in downhill skiing, that the geometric shape of the racing line in the Brüggli-S segment of the Lauberhorn racing track has a large impact on the athlete's success. This knowledge is based on years of experience. Therefore, our result is not totally new. In fact, it underpins today's intuitive knowledge and understanding.

We are well aware that the sample size of 10 is small, and clearly the message of this case study would be much stronger, if we were able to present data for a larger sample size. But, analyzing data with a considerably larger sample size faces severe restrictions:

- the number of world-class downhill skiing athletes is by definition small,
- due to the competitive nature of the sport, it is close to impossible to share data for athletes of different nations, and
- to minimize environmental influences on the data, the measurements have to be taken on a single day over a short period of time.

Therefore, we have chosen to present the data on the small sample size of 10, and to draw conclusions as carefully as possible. For the future, we hope to encourage analogous measurements for other athletes by other researchers worldwide, to further strengthen our conclusions.

In detail, our results apply to this group, and not to athletes with different performance levels, or different materials, or racing lines recorded under different environmental conditions. Clearly, for female athletes or athletes with different skill levels, the results will be quantitatively different. We suggest, however, that the results will also be qualitatively similar for female athletes or athletes of different skill levels. Therefore, the results presented are a case study. Even if the generalization will be extremely interesting on one side, it will be extremely hard on the other side, since for competitive reasons it will be close to impossible to make a similar study with world-class athletes from all nations.

References


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Competing interests
The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Data availability statement
All relevant data are within the paper.

Author Contributions
EK and BB set up and conducted the experiment. MG, LS, MS, AV and MJB processed and analyzed the measured data. All authors contributed to the article and approved the submitted version.

Ethics statement
The studies involving human participants were reviewed and approved by the local institutional ethics committee. The participants provided their written informed consent to participate in this study.